DIGITAL CIRCUITS

In addition to this instructions, please read the following chapters in the book 'Basic electronics for scientists and engineers' by D. L. Eggleston:

8. Digital circuits and devices, and the following chapters:
    - 8.1. Introduction
    - 8.2. Binary numbers
    - 8.3. Representing binary numbers in a circuit
    - 8.4. Logic gates
    - 8.5. Implementing logical functions
    - 8.6. Boolean algebra
    - 8.7. Making logic gates

Additional resources:
https://www.electronics-tutorials.ws/binary/bin_1.html
https://www.electronics-tutorials.ws/logic/logic_2.html
https://www.electronics-tutorials.ws/logic/logic_3.html
https://www.electronics-tutorials.ws/logic/logic_4.html (excluding Schmitt variations)
https://www.electronics-tutorials.ws/logic/logic_5.html
https://www.electronics-tutorials.ws/logic/logic_6.html

Digital systems are binary systems in which devices can operate in only two possible states. Digital circuits process signals that contains only two possible values or states, called logic '0' and logic '1'. On the contrary, analogue circuits process signals that can be of any arbitrary value and can constantly and continuously change from one value to another. Such analogue signals are frequency or amplitude of the voltage and current, and are measured by analogue instruments (e.g. voltmeter).

Basic electronic element in digital circuit must therefore operate in only two possible states, e.g. transistor must operate in two distinctive regions: in saturation region or in cutoff region, but cannot operate in active region where continuous, analogue values and states are found. So, a transistor in a digital circuit will be in saturation where voltage between collector and emitter is high (e.g. 4 ± 1 V), or in cutoff where collector-emitter voltage is low (e.g. 0.2 ± 0.2 V). All other values of the collector-emitter voltage are not possible, and transistors exclusively operates only in these two regions. Consequently, digital states are binary quantised and represent logical states. Different types of representation of logical states in the binary system are used today, as shown in the table below:

First state	true	high	1	up	pulse	on	closed	hot	yes
Second state	false	low	0	down	no pulse	off	open	cold	no

In logic, a statement can be *true* or *false*, in accordance with the first binary representation in the table. A switch in a binary circuit can be *closed* or *open*. Binary logic also means that logical and mathematical operations can be carried out. Therefore, binary arithmetic and mathematical operations of switches or logical functions are best carried out by representing the two possible states as ‘0’ (zero) and ‘1’ (one). These two possible values are also called Boolean values.

Only two valid Boolean values means that the binary system with binary numbers is ideal for use in digital circuits and systems. Any number can be represented in the binary system by expanding into finite power series with the base of 2. In that system, only two digits (0 and 1) are needed to represent any number. In mathematics, we uses decimal system, which is base-10 number system, with 10 possible digits running from 0 to 9. Each number is represented by coefficients in the power series, and in decimal system each following coefficient (digit) from left to right has 100 times less value. Similarly, each following coefficient (digit) from left to right in binary system has 2 times less value.

To summarize, any number *N* can be represented by coefficients (digits) b_i in power expansion series with base *q*:

$$N = \sum_{i=-n}^{n} b_i q^i$$

In decimal system, base is $q = 10$, and in digital system base is $q = 2$. For example, number 193 is represented in decimal and binary system as:

Decimal system: $N = 1 \cdot 10^2 + 9 \cdot 10^1 + 3 \cdot 10^0 = 193$
Binary system: $N = 1 \cdot 2^7 + 1 \cdot 2^6 + 0 \cdot 2^5 + 0 \cdot 2^4 + 0 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0 = 11000001$

Digital values of ‘0’ and ‘1’ are binary digits and they are also called bits (BInary digiTS). In one bit, only two values can be stored (0 or 1), so they are not very useful for information storage and processing. Eight bits can be grouped together to form byte, so one byte can have $2^8 = 256$ possible values. This can be used for storing characters. Modern computers use 4 bytes (32 bits, single precision) or 8 bytes (64 bits, double precision) to store numbers.

DIGITAL LOGIC SYSTEMS

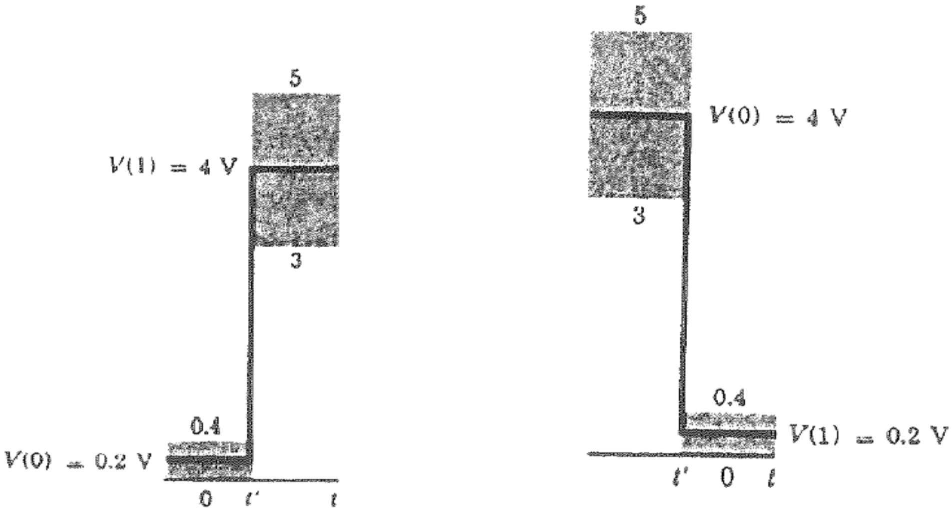
Digital logic is the logic system that determines the manipulation and behaviour of binary values in electronic circuits. Digital logic uses logic gates to perform and implement different logic operations that are the fundamental to any modern computer technology.

Digital logic can be classified into different types in relation to the classification criteria. Two different types of digital logic exists if we take into account the nature of the logic states or values (‘0’ and ‘1’):

1. Logic system of DC levels (level logic system or DC logic)
2. Dynamic logic system or pulse logic system

In **level logic states**, DC voltage is used to represent a level or a logic state. By changing the DC voltage, transit from one logic state to another takes place. In a two-level system, only two values of DC voltage are allowed. The logical value ‘0’ or ‘1’ is related to one of the two possible DC voltage levels. The use of either higher or lower DC voltage level to represent either logical states is fully arbitrary. If the higher DC voltage level corresponds to the logical value ‘1’, and the lower DC voltage level to the logical value ‘0’, then the binary signal representation is called active-high and the logic is called positive logic (figure 1a). On the contrary, if the lower DC voltage level represents the logical state ‘1’, then the binary signal representation is called active-low (figure 1b) and the logic is negative. Absolute values of the DC voltages of the two levels are not important. Logical state ‘0’ doesn’t necessarily correspond to the zero value of the DC voltage.

Characteristics of the real electronic elements can differ greatly from one type of the electronic device to another, and from one manufacturer to another. They can also vary due to the temperature and their use. Instabilities of the power supply and of the electric circuit, as well as the noise generated by different sources in the circuit, can change the value of the DC voltage level. In order to avoid this uncertainty, DC voltage level of each of the two logical states is not strictly determined, but covers a range of voltage values around the given DC voltage. If the voltage is inside that interval, the level is considered to have the corresponding logical value (figure 1). As can be seen from figure 1a, voltage between 3 and 5 V corresponds to the logical value ‘1’, and voltage between 0 and 0.4 V corresponds to the logical value ‘0’. No logical value is associated for voltages between 0.4 and 3 V, and for voltages > 5 V and < 0 V. For example, in CMOS technology (commonly used in cameras, CPUs, computer memories, flash memories,…), low voltage is found between 0 V and 1/3 V_{DD}, and high voltage between 2/3 V_{DD} and V_{DD}, where V_{DD} is the supply voltage. In TTL logic, low voltage covers interval between 0 and 0.8 V, and high voltage between 2 and 5 V.



(a)

(b)

Figure 1. *Logic system of DC levels (level logic system or DC logic): a) active-high (positive logic); b) active-low (negative logic)*

Pulse or dynamic logic system is a logic system where logical value or bit is recognized by the presence or absence of a pulse. If the presence of a positive pulse corresponds to the logical value ‘1’, than the signal representation is called active-high and the logic is positive. On the other hand, if the presence of a negative pulse correspond to the logical value ‘1’, the signal representation is active-low and the logic is negative. In both signal representations, the absence of the pulse corresponds to the logical value ‘0’.

Operation of all digital systems, including the largest and the most complicated systems (computers, supercomputers, data analysis, storage and management, control and communications), are based on binary logic that includes only few basic logic operations. Binary logic is in fact a Boolean logic, which is a two-valued formal logic. Boolean algebra represents a system for mathematical analysis of the logic that includes a set of the basic rules for manipulating the logical values in binary logic. Boolean algebra was introduced by G. Boole in the middle of 19th century. The basic operations of the Boolean algebra, and thus of digital logic are:

a) OR
b) AND
c) NOT

LOGIC GATE (LOGIC CIRCUIT)

Logic gate or logic circuit is an electronic device or electric circuit that implements logic (Boolean) operations. Therefore, logic gates are used to carry out logic operations described by Boolean algebraic equations. By the use of the large number of the logic gates, logic operations can be repeated many times and combined in various ways, so very complex operations can be implemented.

Logic gates (circuits) can be implemented by the use of different electronic devices and elements, such as resistors, transistors, diodes, relays, vacuum tubes, etc. Different types of **logic families** are implemented, depending on the electronic devices and elements used in logic gates. Names of the logic families are derived from the devices and elements used: RTL (resistor-transistor logic), TTL (transistor-transistor logic), DL (diode logic), TDL (tunnel diode logic), DTL (diode-transistor logic), QCA (quantum-dot cellular automata), etc. Semiconductor logic gates are much faster, use less power, are more reliable, can be cascaded and are much smaller than the classical relay and switch equivalents. This enables large number of semiconductor logic gates to be integrated in very small physical space, making integrated circuits (chip or microchip) possible.

Basic logic gates will be described by Boolean algebra and symbols that include:

1. number of inputs and outputs. Inputs are represented by symbols A, B, …, N, while output is represented by the symbol Y. Logic gates with only one output are considered. Inputs and outputs can be in one of the two possible logic states and have logic values 0 or 1.
2. definition of the logic operation
3. standard symbol of the logic gate used in schemes of the digital electric circuits, as well as Boolean equation for the given logic operation
4. truth table, which is an alternative representation of the Boolean (logic) operation. Truth table contains all possible logic values of inputs and corresponding output.
5. Boolean identities or equations that can be verified by the use of the basic definition of the logic operation, of the truth table or of the electronic function of the circuit
6. one of the possible practical realization of the electric circuit.

OR logic gate

1. Input: two or more A, B, ..., N
   Output: single Y
2. Output (Y) is in logic state represented by the value 1 if one or more inputs are in logic state with the value 1
3.

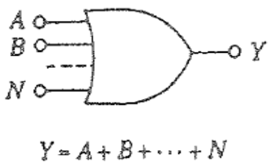


Figure 2. *Symbol of OR logic gate and the corresponding Boolean equation*

4. Truth table

Input		Output
A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Figure 3. *Truth table of OR logic gate*

5. A + B + C = (A + B) + C = A + (B + C)
   A + B = B + A
   A + A = A
   A + 1 = 1
   A + 0 = A

6. Electric circuit scheme

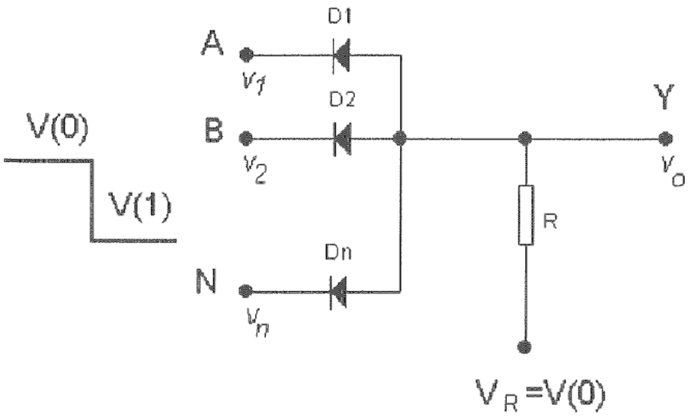


Figure 4. *One of the possible electric circuit realization of OR logic gate in negative DL logic*

Voltage of the power supply V_R is equal to the voltage V(0) which corresponds to logic value 0. Voltage V(0) is higher than the voltage V(1) which corresponds to the logic value 1 in negative logic (active-low). If all the inputs are in logic state 0, with voltage V(0), the voltage drop on all diodes is zero and the diodes are reverse biased. Therefore, there is no current flowing through the diodes and the output voltage is equal to V_R = V(0), which means that the output is in the logic state 0. If one of the inputs, e.g. A, is in the logic state 1 with the voltage V(1) < V(0), the corresponding diode (D_1) becomes forward biased, the current flows through that diode and the voltage drop on D_1 is small (equal to the forward bias voltage ≈ 0.6 V for a silicon diode). Consequently, voltage of the output will be equal to the input voltage V(1), which means that the output is in the logic state 1. All other diodes are reverse biased. We can assume, more or less, that the resistance of the forward biased diode is small, $R_f << R$ and that the voltage drop V_f on the forward biased diode D_1 is negligible compared to the input voltage in A (v_1). In ideal case, we can assume an ideal diode with $R_f = 0$ and $V_f = 0$ so that:

$$v_o = v_1 \qquad \rightarrow \qquad v_o = V(1)$$

In more realistic case:

$$v_o = v_1 - V_f \quad \rightarrow \qquad v_o = V(1) - 0.6\ \mathrm{V}$$

AND logic gate

1. Input: two or more A, B, ..., N
   Output: single Y
2. Output (Y) is in logic state represented by the value 1 only if all inputs are in logic state with the value 1
3.

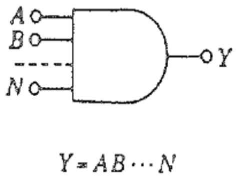


Figure 5. *Symbol of AND logic gate and the corresponding Boolean equation*

4. Truth table

Input		Output
A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

Figure 6. *Truth table of AND logic gate*

5. A · B · C = (A · B) · C = A · (B · C)
   A · B = B · A
   A · A = A
   A · 1 = A
   A · 0 = 0

6. Electric circuit scheme

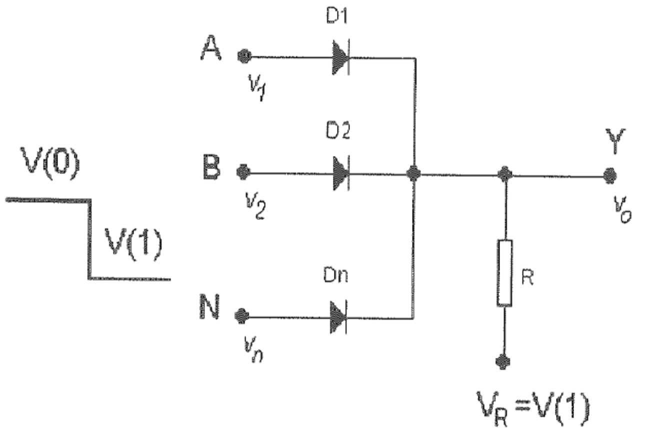


Figure 7. *One of the possible electric circuit realization of AND logic gate in negative DL logic*

Voltage of the power supply V_R is equal to the voltage V(1) which corresponds to logic value 1. If any of the inputs is in logic state 0, with voltage V(0) > V(1), the voltage drop on the diode connected to that input is large enough for the diode to be forward biased. Therefore, the current flows through that diode, the voltage drop is small and the output is almost short-circuited with the input. Consequently, voltage of the output will be equal to the input voltage V(0) on that forward biased diode, which means that the output is in the logic state 0. If all the inputs are in logic state 1 with input voltage V(1), the voltage drop on all diodes is zero, all diodes are reverse biased, and no current flows through them. Consequently, the output voltage will be equal to V_R = V(1), and the output is in logic state 1, Y = 1. The above consideration assumes ideal diodes.

NOT logic gate

1. Input: single A
   Output: single Y
2. Output (Y) is in logic state represented by the value 1 only if the input is not in the logic state with the value 1
3.

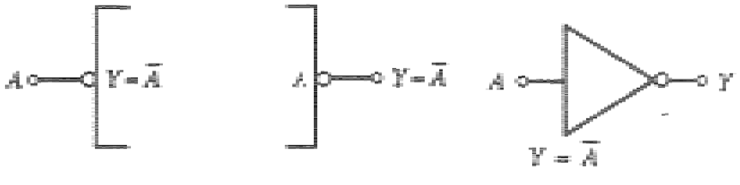


Figure 8. Possible symbols *of NOT logic gate and the corresponding Boolean equation*

4. Truth table

Input	Output
A	Y
0	1
1	0

Figure 9. *Truth table of NOT logic gate*

5. $\overline{\overline{\mathrm{A}}} = \mathrm{A}$
   $\overline{\mathrm{A}} + \mathrm{A} = 1$
   $\overline{\mathrm{A}} \cdot \mathrm{A} = 0$
   $\mathrm{A} + \overline{\mathrm{A}} \cdot \mathrm{B} = \mathrm{A} + \mathrm{B}$

6. Electric circuit scheme

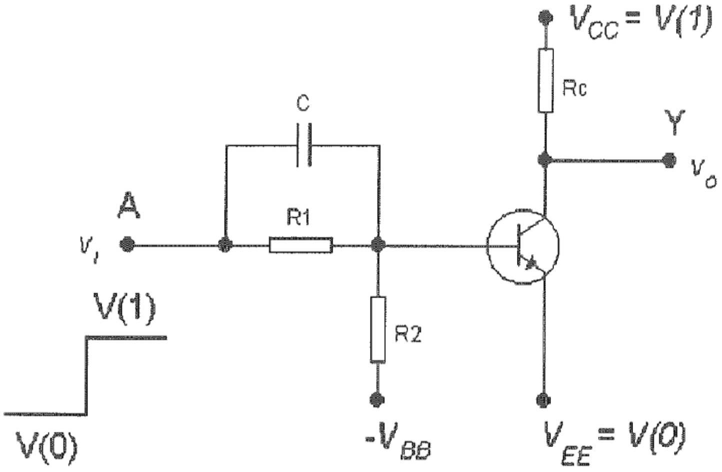


Figure 7. *One of the possible electric circuit realization of NOT logic gate in positive RTL logic*

NOT logic gate is implemented in positive logic, so V(1) > V(0). Voltage V_{EE} = V(0) is applied at the emitter of the transistor, while voltage V_{CC} = V(1) is applied at the collector resistor. If the input voltage is V(0) = V_{EE}, the voltage drop across the base-emitter PN junction is zero, and the base-emitter junction is reverse biased. In that case, transistor is in cut-off region, it is not working, no current flows from emitter to collector, and therefore there is no voltage drop on resistor R_C. Output voltage is:

$$v_0 = I_C R_C + V_{CC}$$

$$I_C \approx 0 \qquad \rightarrow \qquad v_0 \approx V_{CC} = V(1)$$

So, when voltage V(0) is applied at the input, output voltage becomes V(1). Electric circuit is designed so that the transistor is in saturation when voltage V(1) is applied at the input. In this case, voltage V'(1) at the base of the transistor is higher than the voltage at the emitter, V_{EE} = V(0), and the base-emitter PN junction is forward biased. The transistor is in saturation where the current flows through the transistor and $V_{CE} = V_{CEsat} \approx 0$. Due to:

$$v_0 = V_{CE} + V_{EE}$$

output voltage becomes:

$$v_0 \approx V_{EE} = V(0)$$

So, when voltage V(1) is applied at the input, output voltage becomes V(0). In this consideration, we neglected small voltage drop between emitter and collector V_{CEsat} during saturation.

The above three logic gates are the basic logic gates and all other logic operations can be implemented by combining AND, OR and NOT. Two other logic gates commonly used are NAND and NOR which carry out operations that are logically inverse to AND and OR operations, and consists of NOT and OR/AND logical operations.

NOR logic gate

1. Input: two or more A, B, ..., N
   Output: single Y
2. Output (Y) is in logic state represented by the value 1 only if all inputs are in logic state with the value 0
3.

$$Y = \overline{A + B}$$

Figure 11. *Symbol of NOR logic gate and the corresponding Boolean equation*

4. Truth table

Input		Output
A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Figure 12. *Truth table of NOR logic gate*

6. Electric circuit scheme

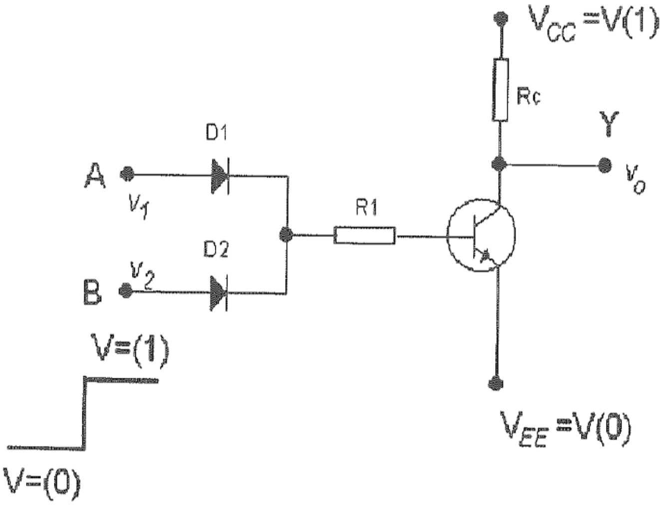


Figure 13. *One of the possible electric circuit realization of NOR logic gate in positive DTL logic*

NOR logic gate is implemented in positive logic, so V(1) > V(0). Voltage V_{EE} = V(0) is applied at the emitter of the transistor, while voltage V_{CC} = V(1) is applied at the collector resistor. If the voltage at one of the inputs (e.g. A) is V(1) > V(0), than the diode at this input (D_1) becomes forward biased, the current flows through that diode, and the voltage drop at the diode is small (≈ 0.6 V). Therefore, this diode becomes almost short-circuited, and the voltage at the base of the transistor becomes equal to the input voltage at forward-biased diode, V(1). Emitter is connected to V_{EE} = V(0), so the voltage drop between base and emitter is V_{BE} = V(1) – V(0) > 0 as V(1) > V(0). Transistor becomes saturated, the current flows through the collector and emitter, and voltage drop between collector and emitter becomes small, $V_{CE} \approx 0$ V. Consequently, output voltage is $v_0 \approx V_{EE}$ = V(0). The same happens if voltage V(1) is applied at both inputs, A and B, so both diodes D_1 and D_2 becomes forward biased. Therefore, the output voltage is again V(0). Contrary to that, if both inputs are at the voltage V(0), the input voltage is not high enough to forward bias any of the diodes, and both diodes D_1 and D_2 are reverse biased. In that case, the voltage at the base of the transistor is small, $V_{BE} \approx 0$, base-emitter PN junction is not forward biased, transistor is in cut-off region, no current flows through the transistor, no voltage drop is present at the resistor R_C and the output voltage is the same as the voltage at the collector, $v_o = V_{CC}$ = V(1).

NAND logic gate

1. Input: two or more A, B, ..., N
   Output: single Y
2. Output (Y) is in logic state represented by the value 1 only if all inputs are not in logic state with the value 0
3.

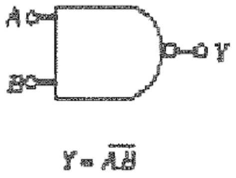


Figure 14. *Symbol of NAND logic gate and the corresponding Boolean equation*

4. Truth table

Input		Output
A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Figure 12. *Truth table of NAND logic gate*

6. Electric circuit scheme

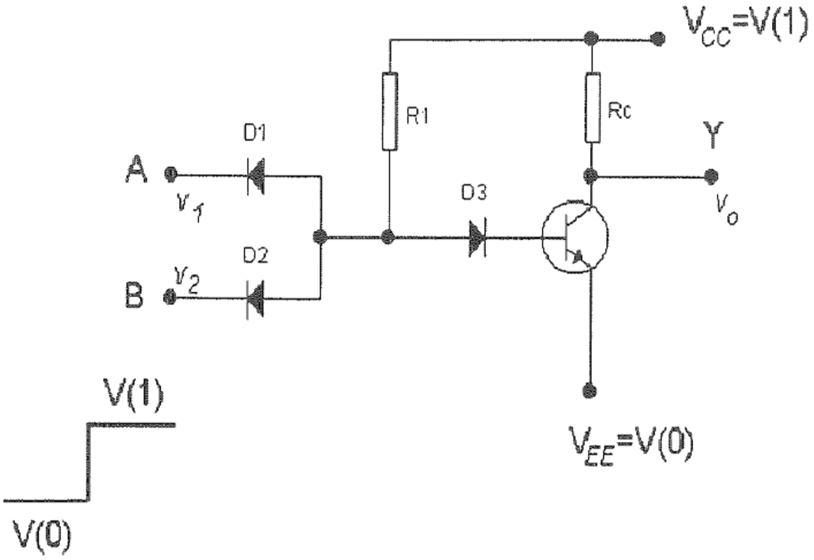


Figure 16. *One of the possible electric circuit realization of NAND logic gate in positive DTL logic*

NAND logic gate is implemented in positive logic, so V(1) > V(0). Voltage V_{EE} = V(0) is applied at the emitter of the transistor, while voltage V_{CC} = V(1) is applied at the collector resistor. If the voltage at both inputs (A and B) are V(1) > V(0), than both diodes D_1 and D_2 are reverse biased, there is no current through any of the two diodes, and they act as disconnected from the rest of the circuit. Diode D_3 is connected to the power supply at the voltage V_{CC} = V(1) across the resistor R_1, and, as V(1) > V(0), diode D_3 is forward biased, the current flows through it and the voltage drop is small. Consequently, voltage at the base of the transistor is:

$$V_B = V_{CC} - V_{D3} - V_{R1}$$

As $V_{D3} \approx 0$ and voltage drop across the resistor R_1 is $V_{R1} < V_{CC}$, voltage at the base becomes:

$$V_B \approx V_{CC} = V(1)$$

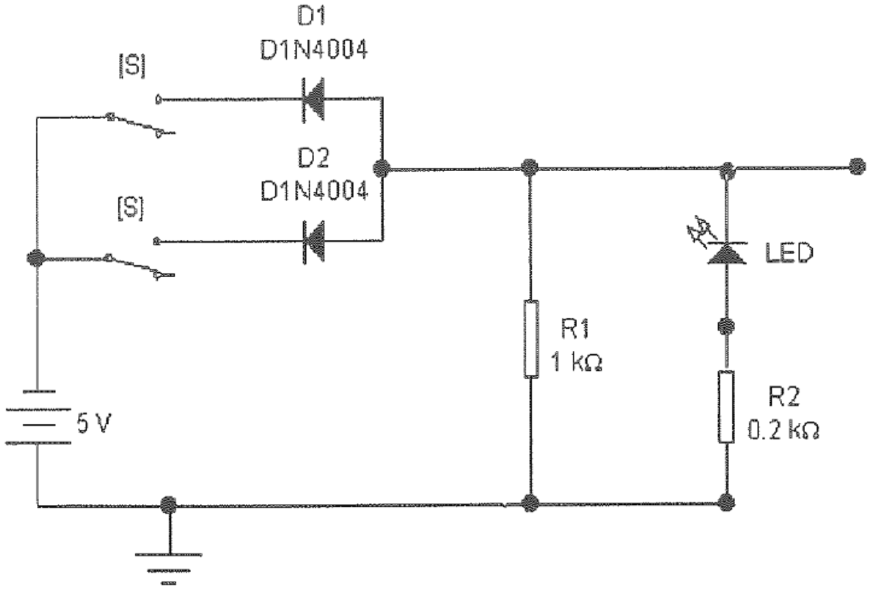
Therefore, voltage at the base, $V_B \approx V(1)$, is larger than the voltage at the emitter, V_{EE} = V(0), the base-emitter PN junction is forward biased and the transistor is in saturation. The current flows through the transistor and voltage between collector and emitter is small, $V_{CE} \approx 0$ V. Consequently, output voltage becomes equal to the emitter voltage V(0), $v_0 \approx V_{EE} = V(0)$.

If only one of the inputs (or both inputs) are at the voltage V(0), the diode connected to this input (or both diodes) is forward biased as its cathode is at the input voltage V(0) and anode is connected to the voltage V(1) > V(0) across the resistor R_1. The current flows through the forward biased diode, its voltage drop is small (≈ 0 V), which means that the anode of the D_3 diode is at the input voltage V(0). Therefore, diode D_3 is reverse biased as the V(0) voltage is not large enough to forward bias that diode. Consequently, base of the transistor is not at the potential large enough to forward bias base-emitter PN junction (emitter is at the voltage V_{EE} = V(0)) and the transistor is in cutoff region (not operating). In cutoff region, there is no current flowing through the transistor, no voltage drop at the resistor R_C, and collector voltage, which is also output voltage, becomes V(1):

$$v_o = V_{CC} - I_C R_C; \qquad I_C \approx 0 \qquad \rightarrow \qquad v_o \approx V_{CC} = V(1)$$

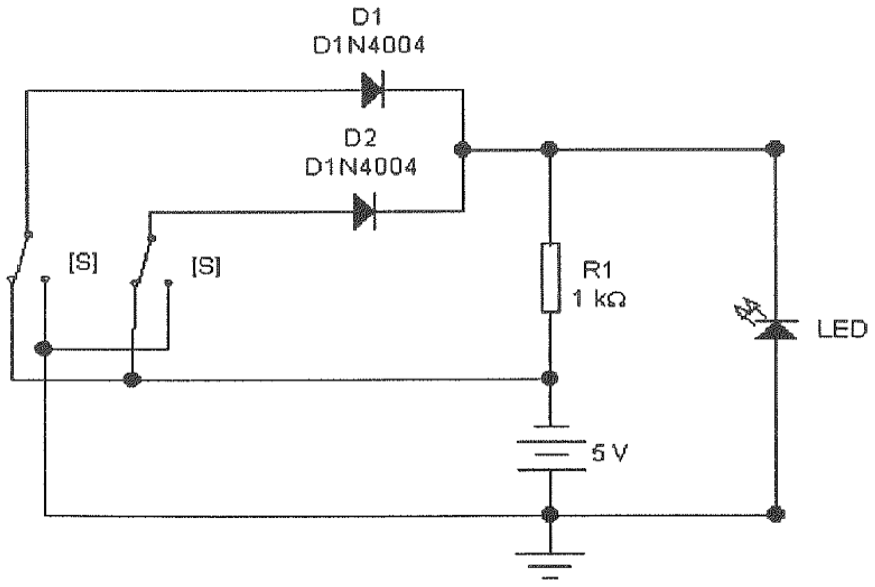
ASSIGNMENT I:

1. Assembly OR logic gate in negative DL logic according to the circuit diagram shown below. Verify the truth table and explain the operation and function of the electric circuit by the use of the switches and LED diode.
2. Measure the voltages at the inputs and output and present them together on V(t) diagrams for every possible combination of input and output values, i.e. so that each diagram represents one row of the truth table.



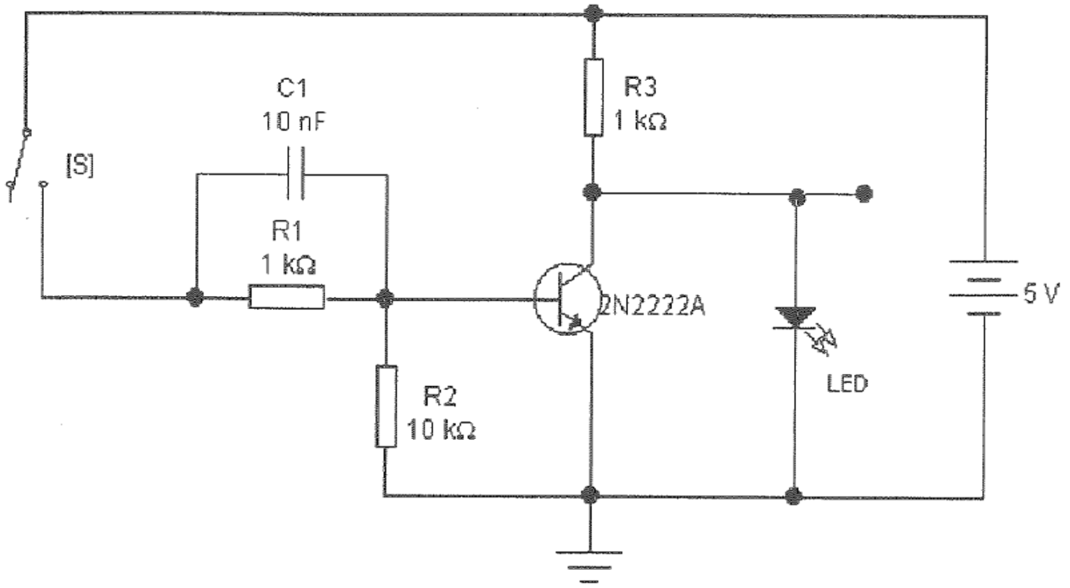
ASSIGNMENT II:

1. Assembly AND logic gate in negative DL logic according to the circuit diagram shown below. Verify the truth table and explain the operation and function of the electric circuit by the use of the switches and LED diode.
2. Measure the voltages at the inputs and output and present them together on V(t) diagrams for every possible combination of input and output values, i.e. so that each diagram represents one row of the truth table.



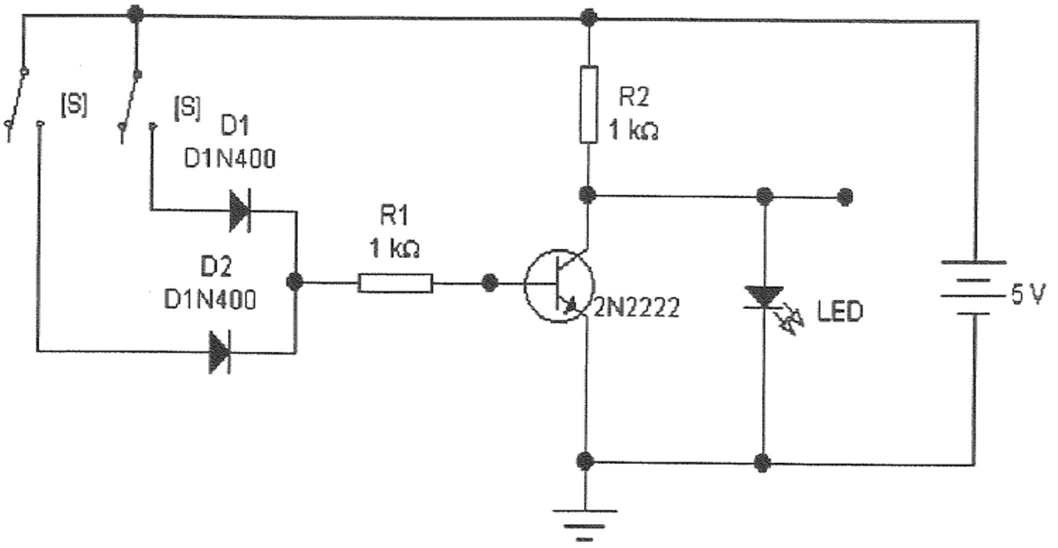
ASSIGNMENT III:

1. Assembly NOT logic gate in positive RTL logic according to the circuit diagram shown below. Verify the truth table and explain the operation and function of the electric circuit by the use of the switches and LED diode.
2. Measure the voltages at the inputs and output and present them together on V(t) diagrams for every possible combination of input and output values, i.e. so that each diagram represents one row of the truth table.



ASSIGNMENT IV:

1. Assembly NOR logic gate in positive DTL logic according to the circuit diagram shown below. Verify the truth table and explain the operation and function of the electric circuit by the use of the switches and LED diode.
2. Measure the voltages at the inputs and output and present them together on V(t) diagrams for every possible combination of input and output values, i.e. so that each diagram represents one row of the truth table.



ASSIGNMENT V:

1. Assembly NAND logic gate in positive DTL logic according to the circuit diagram shown below. Verify the truth table and explain the operation and function of the electric circuit by the use of the switches and LED diode.
2. Measure the voltages at the inputs and output and present them together on V(t) diagrams for every possible combination of input and output values, i.e. so that each diagram represents one row of the truth table.