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11 Spin

11.1 Pretpostavimo da je čestica sa spinom 1/2 u stanju

$$\chi = \frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 2 \end{pmatrix}$$

(a) Koja je vjerojatnost da mjerenjem S_z dobijemo vrijednost $+\hbar/2$, koja da dobijemo $-\hbar/2$?
(b) Koja je vjerojatnost da mjerenjem S_x dobijemo vrijednost $+\hbar/2$, a koja da dobijemo $-\hbar/2$?
(c) Izračunajte prosječnu vrijednost od S_x u stanju χ.

11.2 Spinski dio valne funkcije elektrona glasi

$$\chi = A\begin{pmatrix} 3i \\ 4 \end{pmatrix}$$

(a) Odredite konstantu normalizacije A.
(b) Nađite prosječne vrijednosti od S_x , S_y , S_z u stanju χ.
(c) Nađite $\langle(\Delta S_x)^2\rangle$, $\langle(\Delta S_y)^2\rangle$, $\langle(\Delta S_z)^2\rangle$ za stanje χ.

11.3 (a) Nađite svojstvene vrijednosti i svojstvene vektore za S_y .
(b) Ako mjerimo S_y na čestici u spinskom stanju

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\chi_+ + b\chi_-$$

koje vrijednosti možete dobiti i koje su vjerojatnosti dobivanja tih vrijednosti? Za a i b vrijedi $|a|^2 + |b|^2 = 1$.
(c) Ako mjerimo S_y^2 koje vrijednosti možete dobiti i s kojim vjerojatnostima?

11.4 Definirajmo operatore spuštanja J_- i podizanja J_+ na sljedeći način:

$$J_+ \equiv J_x + iJ_y$$
$$J_- \equiv J_x - iJ_y$$

gdje su J_x i J_y komponente angularnog momenta (orbitalnog, spina ili ukupnog). Koristeći samo osnovne komutacijske relacije za komponente J_i pokažite da vrijedi:
(a) $[\mathbf{J}^2, J_i] = 0$ za $i = 3$
(b) $[J_+, J_-] = 2\hbar J_z$
(c) $[J_z, J_\pm] = \pm\hbar J_\pm$
(d) Zbog (a) vrijede sljedeće relacije $\mathbf{J}^2|a, b\rangle = a|a, b\rangle$ i $J_z|a, b\rangle = b|a, b\rangle$. Ovdje su $\{|a, b\rangle\}$ zajednički svojstveni vektori za $\mathbf{J}^2$ i J_z , a skup $\{a\}$ svojstvene vrijednosti za $\mathbf{J}^2$ i skup $\{b\}$ svojstvene vrijednosti za J_z . Dokažite relaciju

$$J_\pm\,|a, b\rangle = c_\pm\,|a, b \pm \hbar\rangle$$

gdje je $c_\pm$ normalizacijska konstanta. Može se pokazati da $a = j(j + 1)\hbar^2$ i $b = m\hbar$ gdje $j = 0, 1/2, 1, 3/2,\ldots$, a $m = -j, -j + 1,\ldots, 0,\ldots, j - 1, j$.

11.5 Konstruirajte matrice projekcija spina za česticu spina $s = 1$ u bazi $\{|1, -1\rangle, |1, 0\rangle, |1, 1\rangle\}$ što su svojstveni vektori za S_z . Koristite relaciju 12.4(d) u obliku

$$S_\pm\,|s, m_s\rangle = \hbar\sqrt{s(s+1) - m_s(m_s \pm 1)}\,|s, m_s \pm 1\rangle$$

11.6 Za matrice spina 1/2 vrijede antikomutacijske relacije

$$\left\{ S_i , S_j \right\} = \frac{1}{2} \hbar^2 \delta_{ij}$$

u što se možemo uvjeriti direktnim uvršavanjem matrica, zapisanih u bilo kojoj bazi, u gornju relaciju. Pomoću te relacije dokažite da vrijedi

$$\mathbf{S}^2 = \frac{3}{4} \hbar^2$$

gdje je $\mathbf{S}^2 = \mathbf{S} \cdot \mathbf{S} = S_x^2 + S_y^2 + S_z^2$.

11.1

$$\chi = \frac{1}{\sqrt{6}}\begin{pmatrix} 1+i \\ 2 \end{pmatrix}$$

(a) Razvijmo χ po χ_+ i χ_-; to su vlastiti vektori od S_z

$$\chi = c_+\chi_+ + c_-\chi_-$$

$$\chi_+ = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \; ; \; \chi_- = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Treba odrediti $|c_+|^2$ i $|c_-|^2$. Pomnožimo sve s lijeva sa χ_+^+ (transponirano, konjugirano)

$$\chi_+^+\chi = c_+\underbrace{\chi_+^+\chi_+}_{(1\;0)\begin{pmatrix}1\\0\end{pmatrix}=1} + c_-\underbrace{\chi_+^+\chi_-}_{(1\;0)\begin{pmatrix}0\\1\end{pmatrix}=0}$$

$$c_+ = (1\;\;0)\begin{pmatrix} 1+i \\ 2 \end{pmatrix}\cdot\frac{1}{\sqrt{6}} = (1+i)\cdot\frac{1}{\sqrt{6}}$$

$$P_+ = |c_+|^2 = \frac{2}{6} = \frac{1}{3}$$

Vjerojatnost da mjerimo $\hbar/2$ iznosi $1/3$.

$$c_- = (0\;\;1)\begin{pmatrix} 1+i \\ 2 \end{pmatrix}\cdot\frac{1}{\sqrt{6}} = \frac{2}{\sqrt{6}}$$

$$P_- = |c_-|^2 = \frac{4}{6} = \frac{2}{3}$$

Vjerojatnost da mjerimo $-\hbar/2$ je $2/3$.

(b) Razvijmo χ po vlastitim vektorima od S_x. Treba najprije naći vlastite vektore u bazi $\{\chi_+, \chi_-\}$ i vlastite vrijednosti od S_x.

$$S_x \doteq \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Problem vlastitih vrijednosti i vektora za S_x

$$\det(S_x - \lambda 1) = 0 \Rightarrow \det\begin{pmatrix} -\lambda & \hbar/2 \\ \hbar/2 & -\lambda \end{pmatrix} = 0$$

$$\Rightarrow \lambda^2 = \left(\frac{\hbar}{2}\right)^2, \; \lambda_{1,2} = \pm\frac{\hbar}{2}$$

Natrag u matricu $S_x - \lambda 1$

$$\lambda_1 = +\hbar/2 \qquad \begin{pmatrix} -\hbar/2 & \hbar/2 \\ \hbar/2 & -\hbar/2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \Rightarrow \begin{matrix} -\hbar/2\, v_1 + \hbar/2\, v_2 = 0 \\ \hbar/2\, v_1 - \hbar/2\, v_2 = 0 \end{matrix}$$

Dvije jednake jednadžbe. Rješenje:

$$v_1 = 1 \;;\; v_2 = 1$$

$$\lambda_2 = -\hbar/2 \qquad \begin{pmatrix} \hbar/2 & \hbar/2 \\ \hbar/2 & \hbar/2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \Rightarrow \quad v_1 \cdot \frac{\hbar}{2} + v_2 \frac{\hbar}{2} = 0$$

Rješenje:

$$v_1 = 1, \; v_2 = -1$$

U obliku jednostupčanih matrica (normiramo!)

$$|S_x;+\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$|S_x;-\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Razvijamo $|\chi\rangle$

$$|\chi\rangle = C_+|S_x;+\rangle + C_-|S_x;-\rangle \quad / \langle S_x;+|$$

$$\langle S_x;+|\chi\rangle = C_+ \underbrace{\langle S_x;+|S_x;+\rangle}_{=1} + C_- \underbrace{\langle S_x;+|S_x;-\rangle}_{=0}$$

$$C_+ = \langle S_x;+|\chi\rangle = \frac{1}{\sqrt{2}}(1,1)\begin{pmatrix} 1+i \\ 2 \end{pmatrix}\cdot\frac{1}{\sqrt{6}} =$$

$$= \frac{1}{\sqrt{12}}\left[(1+i)+2\right] = \frac{3+i}{\sqrt{12}}$$

$$P_+ = |C_+|^2 = \frac{10}{12} = \frac{5}{6}$$

Vjerojatnost da kad mjerimo S_x dobijemo $+\hbar/2$ je $\frac{5}{6}$.

$$C_- = \langle S_x;-|\chi\rangle = \frac{1}{\sqrt{2}}(1,-1)\begin{pmatrix} 1+i \\ 2 \end{pmatrix}\cdot\frac{1}{\sqrt{6}}$$

$$= \frac{1}{\sqrt{12}}\left[1+i-2\right] = \frac{1}{\sqrt{12}}\left[i-1\right]$$

$$P_- = |C_-|^2 = \frac{2}{12} = \frac{1}{6}$$

Vjerojatnost da kad mjerimo S_x dobijemo $-\hbar/2$ je $\frac{1}{6}$.

(c)

$$\langle S_x \rangle = \chi^{+} S_x \chi = \frac{1}{\sqrt{6}} (1-i, 2) \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \frac{1}{\sqrt{6}} \begin{pmatrix} 1+i \\ 2 \end{pmatrix} =$$

$$= \frac{\hbar}{12} (1-i, 2) \begin{pmatrix} 2 \\ 1+i \end{pmatrix} = \frac{\hbar}{12} (2 - 2i + 2 + 2i)$$

$$= \frac{4\hbar}{12} = \frac{\hbar}{3}$$

Mogli smo iskoristiti i rezultate dobijene pod (b) i

$$S_x |S_x; \pm\rangle = \pm \frac{\hbar}{2} |S_x; \pm\rangle$$

$$\langle S_x \rangle = \langle \chi | S_x | \chi \rangle = \left(c_+^* \langle S_x; +| + c_-^* \langle S_x; -| \right) S_x \left(c_+ |S_x; +\rangle + c_- |S_x; -\rangle \right)$$

$$= \left(c_+^* \langle S_x; +| + c_-^* \langle S_x; -| \right) \left(c_+ \cdot \frac{\hbar}{2} |S_x; +\rangle - c_- \frac{\hbar}{2} |S_x; -\rangle \right)$$

$$= \frac{\hbar}{2} |c_+|^2 - \frac{\hbar}{2} |c_-|^2 = \frac{\hbar}{2} \left(\frac{5}{6} - \frac{1}{6} \right)$$

$$= \frac{\hbar}{3}$$

11.2

(a) Normalizacijska konstanta

$$\chi^{+}\chi = 1$$

$$|A|^2(-3i,4)\begin{pmatrix}3i\\4\end{pmatrix} = |A|^2(9+16) = 25|A|^2 = 1$$

$$A = \frac{1}{5} \quad (\text{biramo da bude realna})$$

(b) Uzmimo S_x

$$\langle S_x\rangle = \chi^{+} S_x \chi = \frac{1}{5}(-3i,4)\cdot\frac{\hbar}{2}\begin{pmatrix}0&1\\1&0\end{pmatrix}\frac{1}{5}\begin{pmatrix}3i\\4\end{pmatrix}$$
$$= \frac{\hbar}{50}(-3i,4)\begin{pmatrix}4\\3i\end{pmatrix} = \frac{\hbar}{50}(-12i+12i) = 0$$

Nađimo $\langle(\Delta S_x)^2\rangle$. Znamo

$$\langle(\Delta S_x)^2\rangle = \langle S_x^2\rangle - \underbrace{\langle S_x\rangle^2}_{=0}$$

Računamo S_x^2

$$S_x^2 = \frac{\hbar^2}{4}\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix} = \frac{\hbar^2}{4}\begin{pmatrix}1&0\\0&1\end{pmatrix} = \frac{\hbar^2}{4}\mathbb{1}$$

Imamo,

$$\langle S_x^2\rangle = \frac{\hbar^2}{4} \quad i \quad \langle(\Delta S_x)^2\rangle = \frac{\hbar^2}{4}$$

PRIMJEDBA:

$$\langle\chi|S_x^2|\chi\rangle = \frac{\hbar^2}{4}$$

za bilo koje stanje $|\chi\rangle$. Slično je i za S_y^2, S_z^2

$$\langle S_y^2\rangle = \frac{\hbar^2}{4}, \quad \langle S_z^2\rangle = \frac{\hbar^2}{4}$$

jer je

$$S_x^2 = \frac{\hbar^2}{4}, \quad S_z^2 = \frac{\hbar^2}{4}$$

11.3 (a)

$$S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

$$\det(S_y - \lambda 1) = 0$$

$$\begin{pmatrix} -\lambda & -i\hbar/2 \\ i\hbar/2 & -\lambda \end{pmatrix} = 0 \Rightarrow \quad \lambda^2 = \frac{\hbar^2}{4}$$

$$\lambda_{1,2} = \pm \frac{\hbar}{2}$$

$$\lambda_1 = \frac{\hbar}{2}; \quad \begin{pmatrix} -\hbar/2 & -i\hbar/2 \\ i\hbar/2 & -\hbar/2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad \Rightarrow \quad -\frac{\hbar}{2}v_1 - i\frac{\hbar}{2}v_2 = 0$$

$$i\frac{\hbar}{2}v_1 - \frac{\hbar}{2}v_2 = 0$$

$$\Rightarrow \quad v_1 = 1;\ v_2 = i$$

$$\lambda_2 = \frac{\hbar}{2}; \quad \begin{pmatrix} \hbar/2 & -i\hbar/2 \\ i\hbar/2 & \hbar/2 \end{pmatrix}\begin{pmatrix} v_1 \\ v_2 \end{pmatrix} = 0 \quad \Rightarrow \quad \frac{\hbar}{2}v_1 - i\frac{\hbar}{2}v_2 = 0$$

$$i\frac{\hbar}{2}v_1 + \frac{\hbar}{2}v_2 = 0$$

$$\Rightarrow \quad v_1 = 1;\ v_2 = -i$$

Vlastiti vektori napisani u bazi $\{|+\rangle, |-\rangle\}$ su

$$|S_y;+\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix}$$

$$|S_y;-\rangle \doteq \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

(b)

$$\chi = \begin{pmatrix} a \\ b \end{pmatrix} = a\chi_+ + b\chi_-$$

Potražimo

$$|\chi\rangle = c_+|S_y;+\rangle + c_-|S_y;-\rangle \quad / \langle S_y;+|$$

$$\langle S_y;+|\chi\rangle = c_+\underbrace{\langle S_y;+|S_y;+\rangle}_{=1} + c_-\underbrace{\langle S_y;+|S_y;-\rangle}_{=0}$$

$$c_+ = \frac{1}{\sqrt{2}}(1,-i)\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{\sqrt{2}}(a - ib)$$

$$P_+ = |c_+|^2 = \frac{1}{2}|a-ib|^2 \qquad a,b \in \mathbb{Z}$$

$$P_- = |c_-| = \frac{1}{2}|a+ib|^2 = \frac{1}{2}(a+ib)^*(a+ib) =$$

$$= \frac{1}{2}(a^* - ib^*)(a+ib) = \frac{1}{2}\left(|a|^2 + ia^*b - iab^* + |b|^2\right)$$

(c) Zbog

$$s_Y^2 = \frac{t_1^2}{4} 1$$

vjerojatnost da dobijemo $t_1^2/4$ je jednaka 1.

11.4 Osnovne komutacijske relacije

$$[J_i, J_j] = i\hbar \sum_k \varepsilon_{ijk} J_k$$

$$J^2 = J_1^2 + J_2^2 + J_3^2 \qquad J_1 \equiv J_x ; \; J_2 \equiv J_y ; \; J_3 \equiv J_z$$

(a)

$$[J^2, J_3] = [J_1^2, J_3] + [J_2^2, J_3] + \underbrace{[J_3^2, J_3]}_{=0}$$

$$[J_1^2, J_3] = [J_1, J_3] J_1 + J_1 [J_1, J_3]$$

$$= \left(-J_2 J_1 + J_1 \cdot (-J_2)\right) i\hbar$$

$$[J_2^2, J_3] = [J_2, J_3] J_2 + J_2 [J_2, J_3]$$

$$= \left(J_1 J_2 + J_2 J_1\right) \cdot i\hbar$$

Sve skupa

$$[J^2, J_3] = 0$$

(b)

$$[J_x + iJ_y, J_x - iJ_y] = \underbrace{[J_x, J_x]}_{=0} + i[J_y, J_x] - i[J_x, J_y] + \underbrace{[J_y, J_y]}_{=0}$$

$$= -2i \underbrace{[J_x, J_y]}_{i\hbar J_z} = 2\hbar J_z$$

(c)

$$[J_z, J_x \pm iJ_y] = [J_z, J_x] \pm i[J_z, J_y]$$

$$= i\hbar J_y \pm i(-i\hbar J_x)$$

$$= \pm\hbar \underbrace{(J_x \pm iJ_y)}_{J_\pm} = \pm\hbar J_\pm$$

(d) Zbog $[J^2, J_z] = 0$ smijemo pisati

$$J^2 |a,b\rangle = a|a,b\rangle$$

$$J_z |a,b\rangle = b|a,b\rangle$$

Pogledajmo

$$J_z(J_\pm|a,b\rangle) = (J_zJ_\pm - J_\pm J_z + J_\pm J_z)|a,b\rangle$$

$$= (\underbrace{[J_z, J_\pm]}_{\pm\hbar J_\pm} + J_\pm J_z)|a,b\rangle$$

$$= \pm\hbar J_\pm|a,b\rangle + J_\pm(\underbrace{J_z|a,b\rangle}_{b|a,b\rangle})$$

$$= (b\pm\hbar)J_\pm|a,b\rangle$$

To znači da je vektor $J_\pm|a,b\rangle$ vlastiti vektor za J_z i ima vlastitu vrijednost $b\pm\hbar$. Stoga ga smijemo označiti ovako:

$$J_\pm|a,b\rangle = c_\pm|a,b\pm\hbar\rangle$$

11.1.5

$s = 1$

$m_s = -1, 0, 1$

Baza je $\{|s, m_s\rangle\} = \{|1,-1\rangle, |1,0\rangle, |1,1\rangle\}$ i to su vlastiti vektori operatora S_z. To znači da je S_z u toj bazi dijagonalan

$$
\begin{array}{c|c}
m_s \rightarrow \quad 1 \quad 0 \quad -1 & m_s \downarrow \\
S_z \doteq \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} & \begin{matrix} 1 \\ 0 \\ -1 \end{matrix}
\end{array}
$$

Ili pomoću bra i ket notacije

$$S_z \doteq \hbar |1\rangle\langle 1| + 0 \cdot |0\rangle\langle 0| - \hbar |-1\rangle\langle -1|$$

gdje smo stavili

$$|1,1\rangle \equiv |1\rangle$$
$$|1,0\rangle \equiv |0\rangle$$
$$|1,-1\rangle \equiv |-1\rangle$$

odnosno zapisali smo samo kvantni broj m_s.

Kako naći S_x i S_y? Iz zadatka 12.4 definirali smo

$$S_+ = S_x + iS_y$$
$$S_- = S_x - iS_y$$

a odavde slijedi

$$S_x = \frac{1}{2}(S_+ + S_-)$$
$$S_y = \frac{1}{2i}(S_+ - S_-)$$

Treba naći S_+ i S_- pomoću

$$S_\pm |s, m_s\rangle = \hbar\sqrt{s(s+1) - m_s(m_s \pm 1)}\, |s, m_s \pm 1\rangle$$

Imamo,

OVAJ KET NE POSTOJI U OVOM PROSTORU!

$$S_+|1,1\rangle = \hbar\sqrt{\underbrace{1\cdot 2 - 1\cdot 2}_{0}}\,|1,2\rangle = 0$$

$$S_+|1,0\rangle = \hbar\sqrt{1\cdot 2 - 0}\,|1,1\rangle = \hbar\sqrt{2}\,|1,1\rangle = \hbar\sqrt{2}\,|1\rangle$$

$$S_+|1,-1\rangle = \hbar\sqrt{1\cdot 2 - 0}\,|1,0\rangle = \hbar\sqrt{2}\,|1,0\rangle = \hbar\sqrt{2}\,|0\rangle$$

$$S_-|1,1\rangle = \hbar\sqrt{1\cdot 2 - 0}\,|1,0\rangle = \hbar\sqrt{2}\,|1,0\rangle = \hbar\sqrt{2}\,|0\rangle$$

$$S_-|1,0\rangle = \hbar\sqrt{1\cdot 2 - 0}\,|1,-1\rangle = \hbar\sqrt{2}\,|1,-1\rangle = \hbar\sqrt{2}\,|-1\rangle$$

$$S_-|1,-1\rangle = 0$$

Pripadni matrični elementi za S_+ i S_- su

redak → ↙ stupac

$$\langle 1|S_+|1\rangle = \langle 0|S_+|1\rangle = \langle -1|S_+|1\rangle = 0$$

$$\langle 0|S_+|0\rangle = \langle -1|S_+|0\rangle = 0 \;;\quad \langle 1|S_+|0\rangle = \hbar\sqrt{2}$$

$$\langle -1|S_+|-1\rangle = \langle 1|S_+|-1\rangle = 0 \;;\quad \langle 0|S_+|-1\rangle = \hbar\sqrt{2}$$

m_s	1	0	-1	m_s
	0	$\sqrt{2}$	0	1
	0	0	$\sqrt{2}$	0
	0	0	0	-1

$$S_+ \doteq \hbar\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}$$

$$\langle 1|S_-|1\rangle = \langle -1|S_-|1\rangle = 0 \;;\quad \langle 0|S_-|1\rangle = \hbar\sqrt{2}$$

$$\langle 1|S_-|0\rangle = \langle 0|S_-|0\rangle = 0 \;;\quad \langle -1|S_-|0\rangle = \hbar\sqrt{2}$$

$$\langle 1|S_-|-1\rangle = \langle 0|S_-|-1\rangle = \langle -1|S_-|-1\rangle = 0$$

m_s	1	0	-1	m_s
	0	0	0	1
	$\sqrt{2}$	0	0	0
	0	$\sqrt{2}$	0	-1

$$S_- \doteq \hbar\begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

$$S_x = \frac{1}{2}(S_+ + S_-) = \frac{\hbar}{2}\left(\begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix}\right)$$

$$= \frac{\hbar}{2}\begin{pmatrix} 0 & \sqrt{2} & 0 \\ \sqrt{2} & 0 & \sqrt{2} \\ 0 & \sqrt{2} & 0 \end{pmatrix}$$

$$S_y = \frac{\hbar}{2i}\begin{pmatrix} 0 & \sqrt{2} & 0 \\ -\sqrt{2} & 0 & \sqrt{2} \\ 0 & -\sqrt{2} & 0 \end{pmatrix}$$

11.6 Provjerimo najprije <u>antikomutacijske</u> relacije (bilo bi zgodno napisati kompjutorski program za provjeru)

$$\{S_i, S_j\} = \frac{1}{2}\hbar^2 \delta_{ij}$$

Uzmimo

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \; ; \; S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_x S_z = \left(\frac{\hbar}{2}\right)^2 \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{\hbar^2}{4} \cdot \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

$$S_z S_x = \frac{\hbar^2}{4}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \frac{\hbar^2}{4}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\{S_x, S_z\} = S_x S_z + S_z S_x = 0$$

Iz antikomutacijskih relacija slijedi

$$\{S_i, S_i\} = 2 S_i^2 = \frac{1}{2}\hbar^2$$

$$S_x^2 = \frac{\hbar^2}{4} \, , \; S_y^2 = \frac{\hbar^2}{4} \; ; \; S_z^2 = \frac{\hbar^2}{4}$$

Imamo,

$$S^2 = S_x^2 + S_y^2 + S_z^2 = \frac{3}{4}\hbar^2$$